25/3/25

Problem 1

(a)

(i) $1-i$

$z = 1-i$ $\quad |z| = \sqrt{1^2+(-1)^2}$

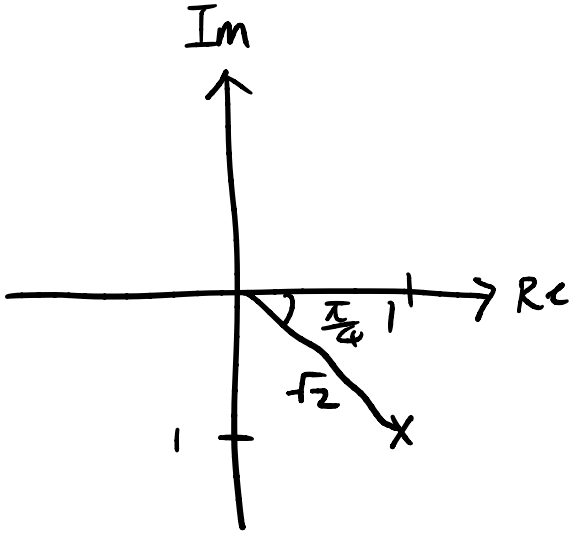
$= \sqrt{2}$

$\text{Arg}(z) = \tan^{-1}\frac{-1}{1}$

$= 2\pi - \frac{\pi}{4}$

$= \frac{7\pi}{4}$

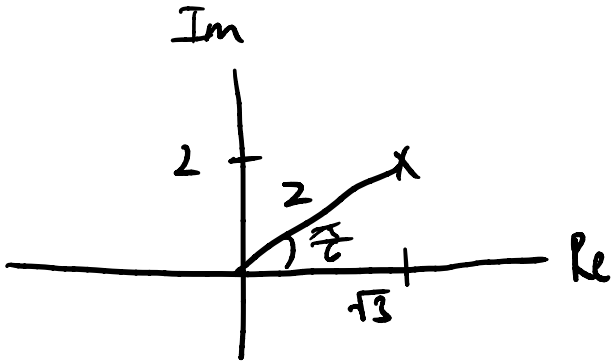
$z = \sqrt{2}e^{\frac{7\pi}{4}i}$



(ii) $z = 2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ $\quad z = 2e^{\frac{\pi}{6}i}$

$= \frac{2\sqrt{3}}{2} + i\frac{2}{2}$

$= \sqrt{3}+i$



(iii) $z = \sqrt{-a-bi}$

(iv) $z = \sqrt[6]{1}$, $0 < \theta < \frac{\pi}{2}$

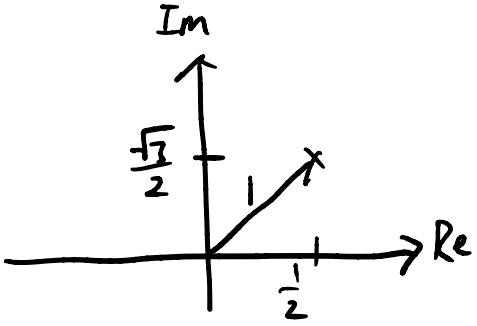
$z^6 = 1$

$z = e^{\frac{2\pi k}{6} i}$, $k = 0, 1, \ldots, 5$

$\Rightarrow z = e^{\frac{\pi}{3} i}$

$= \frac{1+\sqrt{3}i}{2}$

$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$ $\quad z = e^{\frac{\pi}{3} i}$



(v) $z = \left(\frac{1+i}{\sqrt{2}}\right)^{-13}$

$= \left(\frac{\sqrt{2}}{1+i}\right)^{13}$

$= \frac{-\sqrt{2}+\sqrt{2}i}{2}$

$\frac{\sqrt{2}}{1+i} = \frac{\sqrt{2}-\sqrt{2}i}{2}$

$= e^{\frac{\pi}{4} i}$

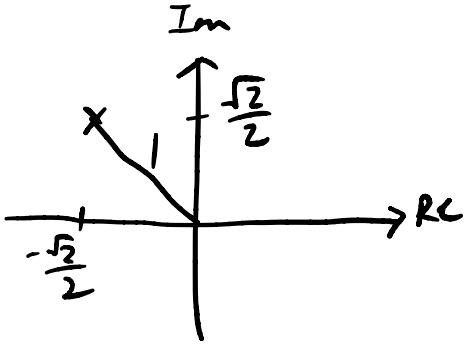
$\left(e^{\frac{\pi}{4} i}\right)^{13} = e^{\frac{13\pi}{4} i}$

$= e^{\frac{3\pi}{4} i}$

$\theta = \tan^{-1} \frac{1}{1}$

$= \frac{\pi}{4}$

$|z| = 1$



(b) $z^4+4=0$

Let $x=z^2$,

then $x^2+4=0$

$$x=\frac{\pm\sqrt{-16}}{2}$$

$$=\pm 2i$$

$$\Rightarrow z^2=2i,\ -2i$$

$$=2e^{\frac{\pi}{2}i},\ 2e^{-\frac{\pi}{2}i}$$

$$z=\sqrt{2}e^{\frac{\pi}{4}i},\ \sqrt{2}e^{-\frac{\pi}{4}i}$$

$$z=e^{\frac{\frac{\pi}{2}+2k\pi}{2}i} \quad \text{where } k=0,1$$

$$=e^{(\frac{\pi}{4}+k\pi)i} \qquad w=e^{ki}$$

$$\Rightarrow z=\sqrt{2}e^{\frac{\pi}{4}i},\ \sqrt{2}e^{\frac{5\pi}{4}i},\ \sqrt{2}e^{-\frac{\pi}{4}i},\ \sqrt{2}e^{\frac{3\pi}{4}i}$$

$$z^2+2z+2=0$$

$$z=\frac{-2\pm\sqrt{4-8}}{2}$$

$$=-1\pm i$$

$$=\sqrt{2}e^{\frac{3\pi}{4}i},\ \sqrt{2}e^{-\frac{3\pi}{4}i}$$